

Level 4: Calculus Blitz – 2008

1. If the tangent line to $y = f(x)$ at $(8, 4)$ passes through $(5, -32)$, find $f'(8)$.

- a) 24 b) 20 c) -12 d) 12 e) None of these

2. Find the domain of the function $f(x) = \frac{1}{1 - e^x}$.

- a) $(-\infty, \infty)$ b) 0 c) $x \neq 0$ d) $(-1, 1)$ e) None of these

3. If an arrow is shot upward on the planet Krypton with velocity of 75 m/s, its height in meters after t seconds is given by $h = 75t - 0.5t^2$. Find the average velocity over the interval $[2, 4]$.

- a) 148 m/s b) 292 m/s c) 144 m/s d) 72 m/s e) None of these

4. $\lim_{x \rightarrow 5} \frac{5 - x}{|5 - x|} =$

- a) Does not exist b) ∞ c) 1 d) -1 e) None of these

5. If $1 \leq f(x) \leq x^2 + 8x + 8$ for all x , find $\lim_{x \rightarrow -1} f(x)$.

- a) Does not exist b) ∞ c) 1 d) 0 e) None of these

6. Determine where f is discontinuous.

$$f(x) = \begin{cases} \sqrt{-x} & \text{if } x < 0 \\ 3 - x & \text{if } 0 \leq x \leq 3 \\ (3 - x)^2 & \text{if } x > 3 \end{cases}$$

- a) 0 and 3 b) 0 only c) 3 only d) -3 e) None of these

7. $\lim_{t \rightarrow \infty} \frac{t^2 + 3}{t^3 + t^2 - 1} =$

- a) ∞ b) 0 c) -3 d) 3 e) None of these

8. If $\lim_{x \rightarrow 7} f(x) = -3$ and $\lim_{x \rightarrow 7} g(x) = 9$, then $\lim_{x \rightarrow 7} \frac{2f(x)}{g(x) - f(x)} =$

- a) 1 b) -1 c) Does not exist d) ∞ e) None of these

9. A rectangle is inscribed in a circle of diameter 1. Find the maximum area of the rectangle.

- a) 1 b) 1/2 c) 1/4 d) $2\sqrt{2}$ e) None of these

10. Let $f(x) = \frac{e^x}{x^2}$. Then $f'(1) =$

- a) 0 b) 1 c) -e d) e e) None of these

11. The gas law for an ideal gas at absolute temperature T (in Kelvins), pressure P (in atmospheres), and volume V (in liters) is $PV = nRT$, where n is the number of moles of the gas and $R = \alpha$ is the gas constant. Suppose that, at a certain instant, $P = 7$ atm and is increasing at a rate of 0.10 atm/min and $V = 10$ L and is decreasing at a rate of 0.15 L/min. Find the rate of change of T with respect to time at that instant if $n = 10$ moles.

- a) $-0.5 / \alpha$ b) $-1.005 / \alpha$ c) $-1.5 / \alpha$
d) $-0.005 / \alpha$ e) None of these

12. Find the derivative of $f(x) = \sqrt{x + \sqrt{x}}$ at $x = 1$.

- a) $\frac{\sqrt{2}}{8}$ b) $\frac{\sqrt{2}}{4}$ c) $\frac{3\sqrt{2}}{8}$ d) $\frac{3\sqrt{2}}{4}$ e) None of these

13. Find $y'(x)$, if $xy^4 + x^2y = x + 3y$.

a) $y' = \frac{-y^4 - 2xy}{4xy^3 + x^2}$

b) $y' = \frac{1 - y^4 - 2xy}{4xy^3 + x^2 - 3}$

c) $y' = \frac{xy^3 + 2x - 3}{x^2y^2(4x - 1)}$

d) $y' = \frac{1 - y^4 - 2x^2}{4xy^3 + x^2 - 3}$

e) None of these

14. If $y = \sqrt{x} \cos \sqrt{x}$, then $y' =$

a) $\cos \sqrt{x} - \frac{\sin \sqrt{x}}{2\sqrt{x}}$

b) $\frac{\sin \sqrt{x} - \sqrt{x} \cos \sqrt{x}}{2\sqrt{x}}$

c) $-\frac{1}{2} \left[\frac{\cos \sqrt{x} - 1}{\sqrt{x}} \right]$

d) $-\frac{1}{2} \left[\frac{\sin \sqrt{x} - 1}{\sqrt{x}} \right]$

e) None of these

15. If $y = x^{6x}$, then $y' =$

a) $6x^{6x}(6 \ln x + 1)$

b) $6x^{6x}(\ln x + 1)$

c) $6(\ln x + 1)$

d) $-6x^{6x}(\ln x + 6)$

e) None of these

16. A lighthouse in the sea is 5 km away from the nearest point A on the seashore. The light is turning clockwise at a speed of 2 revolutions/min. It is currently spotting a point on the shore that is 6 km away from A. Find the speed that the light is moving away from A on the shore in km/min (assuming the seashore is a straight line).

a) Cannot be found

b) $\frac{22\sqrt{6}\pi}{5}$

c) $22\pi/5$

d) $4\sqrt{6}1\pi$

e) None of these

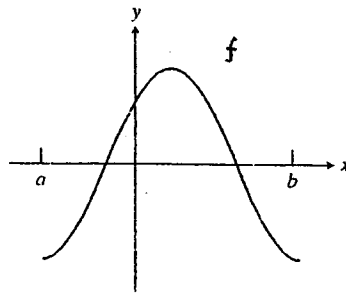
17. A baseball diamond is a square with side 90 ft. A batter hits the ball and runs toward first base with a speed of 26 ft/s. At what rate is his distance from second base decreasing when he is halfway to first base?

- a) $12/\sqrt{5}$ ft/s
 d) $3\sqrt{5}/5$ ft/s

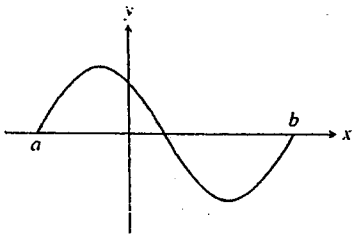
- b) $90/\sqrt{5}$ ft/s
 e) None of these

- c) $26/\sqrt{5}$ ft/s

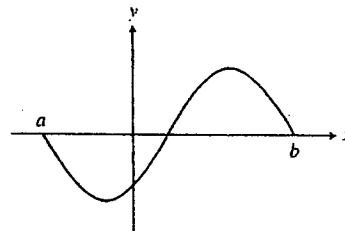
18. The graph of f is shown in the figure below. Which of the following could be the graph of the derivative of f ?



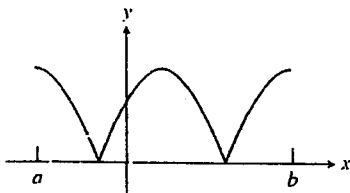
a)



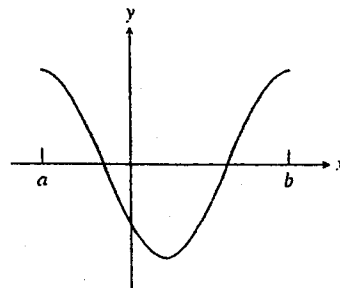
b)



c)



d)



e) None of these

19. If $f'(t) = 2t - 3\sin t$, $f(0) = 5$. Then $f(t) =$

- a) $t^2 + 3\cos t + 2$ b) $t^2 + 3\cos t$ c) $2t - 3\sin t + 5$
d) $t^2 - 3\cos t - 5$ e) None of these

20. Let $f(x) = x^3 - 21x^2 + 80x + 7$. Find all numbers c that satisfy the conclusion of Rolle's Theorem on the interval $[0, 16]$.

- a) $201 + \frac{\sqrt{7}}{3}$ b) $7 - \frac{\sqrt{201}}{3}$ c) $201 - \frac{\sqrt{7}}{3}$ and $201 + \frac{\sqrt{7}}{3}$
d) $\frac{7 + \sqrt{201}}{3}$ and $\frac{7 - \sqrt{201}}{3}$ e) None of these

21. Find the maximum area of a rectangle that can be **circumscribed** about a given rectangle with length $L = 8$ and width $W = 3$.

- a) 48.5 b) 72.5 c) 60.5 d) 65.5 e) None of these

22. Find two positive numbers whose product is 196 and whose sum is a minimum.

- a) 4, 48 b) 2, 98 c) 14, 14 d) Cannot be found
e) None of these

23. Evaluate the definite integral $\int_{-\pi/8}^{\pi/8} \frac{x^2 \sin x}{5 + x^6} dx$.

- a) Cannot be found b) 1 c) 3 d) 0 e) None of these

24. Find the derivative of the function $y = \int_{\sqrt{x}}^x \frac{e^t}{t} dt$.

a) $\frac{xe - e^{\sqrt{x}}}{x}$

b) $\frac{2e^x - e^{\sqrt{x}}}{2x}$

c) $\frac{e^x - 2\sqrt{x}(e^x)}{x}$

d) $\frac{e^x}{x}$

e) None of these

25. Evaluate the integral $\int_1^4 \frac{x^2 + 6}{\sqrt{x}} dx$.

a) 24.4

b) 37

c) 74

d) 49.2

e) None of these

26. Evaluate the integral $\int_{e^{16}}^{e^{64}} \frac{dx}{x\sqrt{\ln(x)}}$.

a) 6

b) 8

c) 9

d) 11

e) None of these

27. Evaluate the integral $\int_0^1 \frac{e^x}{1+e^{2x}} dx$.

a) $\arctan e$

b) $\pi/4$

c) $\frac{\pi}{4} \arctan e$

d) $\arctan e - \frac{\pi}{4}$

e) None of these

28. The area of the region bounded by the curves $y = x^2 - 2x$ and $y = x + 4$ is:

a) $\frac{125}{3}$

b) $\frac{25}{3}$

c) 5

d) 20

e) None of these

29. Evaluate $\lim_{n \rightarrow \infty} \frac{1}{n} \left[\left(\frac{1}{n}\right)^9 + \left(\frac{2}{n}\right)^9 + \left(\frac{3}{n}\right)^9 + \dots + \left(\frac{n}{n}\right)^9 \right]$

- a) Cannot be found b) 1 c) 1/2 d) 1/10 e) None of these

30. If $f(x) = 2x^3 + \frac{1}{x^2} + 16x^{7/2}$, then $f''(x) =$

- a) $12x + \frac{6}{x^4} + 140x^{3/2}$ b) $6x^2 - \frac{2}{x^3} + 56x^{5/2}$
c) $x^2 + \frac{6}{x^4} - 140x^{3/2}$ d) $12x - \frac{6}{x^4} + 56x^{5/2}$ e) None of these

31. Find all of the inflection point(s) on the graph of $f(x) = 4x^{1/3} + x^{4/3}$.

- a) $(-1, -3)$ b) $(0, 0)$ c) $(-1, -3)$ and $(2, 6\sqrt[3]{2})$
d) $(0, 0)$ and $(2, 6\sqrt[3]{2})$ e) None of these

32. Let $x^3 + y^3 = 3xy$. The slope of the tangent line at $\left(\frac{2}{3}, \frac{4}{3}\right)$ is:

- a) Cannot be found b) -1 c) $\frac{3}{4}$ d) $\frac{4}{5}$ e) None of these

33. $\lim_{n \rightarrow \infty} \frac{1^2 + 2^2 + 3^2 + \dots + n^2}{n^3} =$

- a) ∞ b) $\frac{1}{3}$ c) 0 d) Cannot be found e) None of these

34. The average value of $f(x) = \frac{1}{\sqrt{x}}$ on the interval $[1, 4]$ is
- a) $\frac{1}{3}$ b) $\frac{2}{3}$ c) 1 d) $\frac{3}{2}$ e) None of these

35. Use the method of cross sections to find the volume of the solid obtained by rotating the region bounded by the graphs of $y = \frac{1}{x}$, $y = 0$, $x = 0.1$, and $x = 2$ about the x -axis.

- a) 7π b) 8π c) 9π d) 10π e) None of these

36. Use the method of cylindrical shells to find the volume of the solid obtained by rotating the region bounded by the graphs of $y^2 = x$ and $y = x^3$ about the y -axis.

- a) $\frac{\pi}{5}$ b) $\frac{2\pi}{5}$ c) $\frac{3\pi}{5}$ d) $\frac{4\pi}{5}$ e) None of these

37. Compute $\lim_{h \rightarrow 0} \frac{\tan\left(\frac{\pi}{4} + h\right) - 1}{h}$.

- a) 2 b) $\frac{\sqrt{2}}{2}$ c) $\frac{1}{2}$ d) $\sqrt{2}$ e) None of these

38. If $\frac{dy}{dt} = my$ and m is a nonzero constant, then y could be

- a) $e^{mt} + 4$ b) $4e^{mt}$ c) $my + 4$
d) $\frac{m}{2}y^2 + 4$ e) None of these

39. $\int \frac{\sin 2x}{1 + \cos^4 x} dx =$

a) $1 - \cos^2 x + c$

b) $-\arctan(\cos^2 x) + c$

c) $-\arcsin(\tan x) + c$

d) $-\cos^2 2x + c$

e) None of these

40. If f is a linear function and $0 < a < b$, then $\int_b^a f''(x) dx =$

a) 0

b) 2

c) $\frac{a^2 - b^2}{2}$

d) $m(a - b)$

e) None of these