2008 BOMB – Page 1

School_____ Team_____

Team Members 1._____ 2._____

3._____ 4._____ 5._____

Instructions: Label answers with appropriate units.
Do not round or approximate answers.
Write answers on the lines provided.

1. If half a chicken can lay half an egg in half a day, how many eggs can 15 chickens lay in 15 days?

Ans:_____

2. For a sequence $\{a_n\}$, the sum of the first n terms is defined as $S_n = 2n^2 - 14n$. Find the value of a_{27}.

Ans:_____

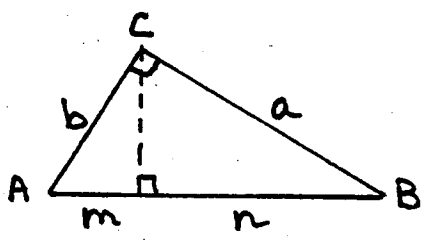
3. Evaluate: $\lim_{x \to 1^+} \frac{\sqrt{x^2-1}}{\sqrt[3]{x-1}}$

Ans:_____

4. Solve: $\sin^2(2x) + \sin^2 x = 1,\ 0 \le x < 2\pi$

Ans:_____

5. In the right triangle ABC, $m + n = 3b$. (See figure.) Find the numerical value of the ratio $\frac{m}{n}$.



Ans:_____

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Instructions: Label answers with appropriate units.
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1. A swimming pool contains x gallons of water. On the first day, 25 gallons of water are added to the pool. Every successive day, 2 gallons more are added than on the previous day. Every day, 50 gallons are pumped out. After how many days will the pool contain the least amount of water?

Ans:_____

2. If $\log_b 2 = \frac{1}{3}$ for $b > 0$, find $\log_2 \frac{b}{\sqrt{8}}$.

Ans:_____

3. Find the slope of each line that goes through the point $(2,-3)$ and is tangent to the parabola $y = x^2 + x$.

Ans:_____

4. Find the area of a regular hexagon inscribed in a circle with diameter 8 inches.

Ans:_____

5. Three hours from now it will be half as long until midnight as it would be one hour from now. What time is it?

Ans:_____

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Instructions: Label answers with appropriate units.
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1. Solve: $x^{\log x^2} = 10^2$

Ans:_____

2. Find all values of m so that the equation $(m^2-3)x^2-15x+2m=0$ has two distinct solutions which are reciprocals of each other.

Ans:_____

3. A bag contains 90 marbles: 35 green, 25 yellow, 25 red, plus a mix of 5 white and blue marbles. What is the smallest number of marbles you have to take out of the bag (without looking!) to be sure that you have 10 marbles of the same color?

Ans:_____

4. In triangle ABC, D is any point on side AB, and E and F are the midpoints of AC and BC respectively. Find the ratio of the area of the quadrilateral $DECF$ to the area of the triangle ABC.

Ans:_____

5. A right circular cylinder is inscribed in a sphere of radius R. Find the largest possible volume of such a cylinder.

Ans:_____

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1. Express $\frac{1}{\sqrt[3]{2}+\sqrt[3]{3}}$ in the form $a\sqrt[3]{4}+b\sqrt[3]{6}+c\sqrt[3]{9}$ where a, b, and c are rational numbers.

Ans:_____

2. Evaluate $\sin x \cos x$ if $\sin x + \cos x = \frac{3}{4}$

Ans:_____

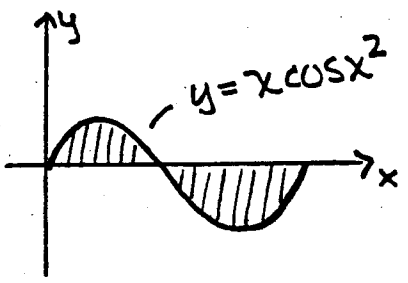
3. A circle is inscribed in a rhombus. Find the area of the rhombus if its acute angles are 30° and the radius of the circle is 10 cm.

Ans:_____

4. At 60 miles per hour, it takes 60 seconds to travel a mile. At 120 miles per hour, it takes 30 seconds to travel a mile. At what speed would it take 45 seconds to travel a mile?

Ans:_____

5. Find the total area of the shaded region.



Ans:_____

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1. The base of a solid S is a triangular region with vertices $(0,0)$, $(3,0)$ and $(0,2)$. Cross sections of S perpendicular to the y-axis are semicircles. Find the volume of S.

Ans:_____

2. What is the final digit of 2^{1423}?

Ans:_____

3. If each of three semicircles has a radius of 4, find the area of the shaded region.

Ans:_____

4. Evaluate: $\sin\left[\sin^{-1}x+\cos^{-1}x\right]$ for $0\le x\le 1$

Ans:_____

5. Solve: $x^2-x+\sqrt{x^2-x+3}-9=0$

Ans:_____