Level 4 (Calculus) Blitz – 2007

1. An open-topped box is to be constructed from a square piece of cardboard with edge length 50 in. First, four (4) squares each having edge length x in. are cut from the four corners of the board. The remaining flaps are folded up. The volume V of the box (in cubic inches) as a function of x is $V(x) =$

a) $x(50-x)^2$ b) $4x(50-x)^2$ c) $4x(50-2x)^2$
d) $4x(25-x)^2$ e) None of these

2. $\lim_{x\to 0} \frac{x-1}{|x-1|} =$

a) Does not exist b) -1 c) 1 d) 0 e) None of these

3. $\lim_{x\to 1}\left(2x^2-3x+4\right) =$

a) -1 b) 3 c) -3 d) 1 e) None of these

4. $\lim_{x\to 9} \frac{3-\sqrt{x}}{9-x} =$

a) Does not exist b) 6 c)3 d) -3 e) None of these

5. $\lim_{x\to -4} \sqrt[3]{(x+1)^6} =$

a) Does not exist b) 9 c) 3 d) $\sqrt{3}/2$ e) None of these

6. $\lim_{x\to 0} \frac{\sqrt{x+4}-2}{x} =$

a) Does not exist b) 6 c) 1/4 d) 4 e) None of these

7. $\lim_{x\to 0} \frac{1}{x}\left(\frac{1}{x+5}-\frac{1}{5}\right) =$

a) $-1/25$ b) Does not exist c) ∞ d) 1/5
e) None of these

8. $\lim_{x\to 5^-} \dfrac{x-5}{|x-5|} =$

a) 1 b) -1 c) Does not exist d) ∞ e) None of these

9. Find the derivative of $f(x) = \dfrac{1}{2x-1}$

a) $-\dfrac{2}{(2x-1)^2}$ b) $-\dfrac{1}{(2x-1)^2}$ c) $\dfrac{2}{(2x-1)^2}$ d) $\dfrac{1}{(2x-1)^2}$

e) None of these

10. The function $f(x) = \dfrac{x-1}{x^3-1}$

a) is a continuous function on $(-\infty, \infty)$. b) has a jump at $x = 1$.

c) has a removable discontinuity at $x = 1$. d) goes to infinity at $x = 1$.

e) None of these

11. $\dfrac{d}{dx}\left(x^2 + \dfrac{3}{x^2}\right) =$

a) $2x^2 - \dfrac{3}{x^3}$ b) $2x - \dfrac{6}{x^3}$ c) $2x - \dfrac{3}{x^3}$ d) $2x^2 - \dfrac{6}{x^3}$

e) None of these

12. If $x^2y^2 = x + y$, then $\dfrac{dy}{dx} =$

a) Cannot be found b) $\dfrac{2xy^2 - 1}{2x^2y - 1}$ c) $4xy - 1$ d) $\dfrac{2xy^2 - 1}{1 - 2x^2y}$

e) None of these

13. Find $\dfrac{dy}{dx}$ if $y = \dfrac{x + \sin x}{x^2 + \cos x}$

a) $\dfrac{x + x^2 - x\sin x + x^2\cos x}{x^2 + \cos x}$ b) $\dfrac{1 - x^2 - x\sin x + \cos x(1 + x^2)}{(x^2 + \cos x)^2}$

c) $\dfrac{x^2 + x\sin x + x^2\cos x}{(x^2 + \cos x)^2}$ d) $\dfrac{1 + \cos x}{2x - \sin x}$ e) None of these

14. $\lim_{x \to 0+} \sqrt{x} \sin \frac{1}{x} =$

a) 0 b) Does not exist c) 1 d) ∞ e) None of these

15. Find $\frac{d}{dx} h(x)$ if $h(x) = \sqrt[3]{x + x^4}$

a) $\frac{1}{3}(x + x^4)^{-2/3}$ b) $\frac{1}{3}(1 + 4x^3)^{-2/3}$

c) $3(x + x^4)^{-2/3}(1 + 4x^3)$ d) $3(1 + 4x^3)^{-2/3}$ e) None of these

16. $\lim_{x \to \infty} \left(1 + \frac{2}{x}\right)^x =$

a) ∞ b) 1 c) e^2 d) Does not exist e) None of these

17. The range of a projectile with initial velocity v is given by

$$R = \frac{v^2 \sqrt{2}}{16}(\cos\theta \sin\theta - \cos^2\theta)$$

where θ is the initial elevation of the projectile. What value of θ maximizes R?

a) Cannot be found b) $\theta = \pi/4$ c) $\theta = 3\pi/8$

d) $\theta = \pi/3$ e) None of these

18. Two vertices of a trapezoid are at (–2, 0) and (2, 0) and the other two lie on the semicircle $x^2 + y^2 = 4$, $y \geq 0$. What is the maximum possible area of the trapezoid?

a) $\frac{4}{\sqrt{5}}\left(2 + \frac{2}{\sqrt{5}}\right)$ b) $\sqrt{3}/3$ c) $3\sqrt{3}$ d) $4\sqrt{3}$ e) None of these

19. If $y = \sin(x^2)$, then dy in terms of dx is:

a) $\cos(x^2)\,dx$ b) $2x\cos(x^2)\,dx$ c) $\cos(2x)\,dx$

d) Cannot be found e) None of these

20. If $x^{1/3}+y^{1/3}=1$, then $\dfrac{d^2y}{dx^2}$ =

a) Cannot be found b) $\frac{2}{3}x^{-1/3}y^{2/3}$ c) $\frac{2}{3}x^{-5/3}y^{1/3}$

d) $\frac{2}{3}x^{-4/3}(1+y^{1/3})$ e) None of these

21. If $\sin(xy)=xy$, then $dy/dx=$

a) $\dfrac{y[1-\cos(xy)]}{x[\cos(xy)-1]}$ b) $\dfrac{\cos(y+x)}{y+x}$ c) $\dfrac{\cos(xy)+y}{\cos(xy)+x}$

d) $\dfrac{xy\cos(xy)}{y+x}$ e) None of these

22. The absolute minimum value of $f(x)=x+\dfrac{4}{x}$ for $0<x<\infty$ is:

a) 2 b) 3 c) 4 d) 5 e) None of these

23. The equation $2x^5+5x^4=4$, has at most how many real solution(s)?

a) 1 b) 2 c) 3 d) 4 e) None of these

24. Find the reason(s) that the function $f(x)=1-|x|$ does not satisfy the conditions of Rolle's Theorem on [1, 3].

a) f is not continuous on [1, 3] b) f is not differentiable on (1, 3)

c) $f(1)\neq f(3)$ d) both (b) and (c), e) None of these

25. The inflection point(s) of $f(x)=4x^{1/3}+x^{4/3}$ is

a) (–1, –3) b) (0, 0) c) $(2, 6\sqrt[3]{2})$ d) both (b) and (c)

e) None of these

26. $\lim\limits_{x\to\infty}\dfrac{2+x-x^2}{(x-1)^2}=$

a) ∞ b) 1 c) – 1 d) Cannot be found e) None of these

27. The asymptotes of the graph of $f(x)=\dfrac{x^2+x-1}{x-1}$ are:

a) the vertical line $x=1$ b) the horizontal line $y=1$
c) the line $y=x+2$ d) (a) and (c) e) None of these

28. The most general antiderivative of $g(x)=\sqrt[3]{x^2}+\dfrac{4}{\sqrt[4]{x^5}}$ is:

a) $\frac{3}{5}x^{5/3}-16x^{-1/4}$ b) $\frac{3}{5}x^{5/3}+16x^{-1/4}$
c) $\frac{3}{5}x^{5/3}-16x^{-1/4}+c$ d) Does not exist e) None of these

29. Find a function $y=f(x)$ that satisfies the following conditions:
A: $\dfrac{dy}{dx}=3x^2+2x$ **and** **B:** $f(0)=5$.

a) x^3+x^2 b) x^3+5 c) x^3+x^2+5 d) $2x^3+x^2+5$
e) None of these

30. If $F(x)=\int_2^x \cos(t^2)\,dt$, then $F'(x)=$

a) Cannot be found b) $2x\sin(x^2)$ c) $-\cos(x^2)$
d) $-\sin(x^2)$ e) None of these

31. If $F(x)=\int_1^{x^2} \sin t\,dt$, then $F'(x)=$

a) $\cos(x^2)$ b) $\sin(x^2)$ c) $2x\sin(x^2)$
d) $2x\cos(x^2)$ e) None of these

32. $\int_1^9 (\sqrt{x}-\dfrac{2}{\sqrt{x}})\,dx=$

a) $\frac{28}{3}$ b) $\frac{14}{3}$ c) $\frac{25}{3}$ d) $\frac{22}{3}$ e) None of these

33. Evaluate $\int_1^3 (1+2x-4x^3)\,dx$

a) 70 b) 88 c) -88 d) -70 e) None of these

34. $\int_1^3 (x-1)^5\,dx =$

a) $\frac{29}{3}$ b) $\frac{32}{3}$ c) $\frac{31}{3}$ d) $\frac{34}{3}$ e) None of these

35. $\int \sin\frac{t}{3}\,dt =$

a) $3\cos\frac{t}{3}+c$ b) $\frac{1}{3}\cos\frac{t}{3}+c$ c) $3\sin\frac{t}{3}+c$

d) $-3\cos\frac{t}{3}+c$ e) None of these

36. Find the area that enclosed by the curves $y=6-x^2$ and $y=x$.

a) $\frac{82}{3}$ b) $\frac{179}{6}$ c) $\frac{123}{4}$ d) $\frac{123}{5}$ e) None of these

37. Find the volume of the solid that is generated by rotating the plane region bounded by $y=x^2$, $y=4$ about the $x-$axis.

a) $\frac{256\pi}{5}$ b) $\frac{128\pi}{5}$ c) $\frac{64\pi}{5}$ d) $\frac{121\pi}{3}$ e) None of these

38. The plane region bounded by $y=25-x^2$ and $y=0$ is rotated about the $y-$axis. Find the volume of the resulting solid.

a) 125π b) 312.5π c) 36π d) 39π e) None of these

39. Find the volume of the solid obtained by rotating the region bounded by $y=x-x^2$ and $y=0$ about the line $x=2$.

a) $\pi/2$ b) $\pi/3$ c) $2\pi/3$ d) $2\pi/5$ e) None of these

40. A particle is moved along the $x-$axis by a force that measures $\frac{10}{(1+x)^2}$ pounds at a point x from the origin. Find the work done in moving the particle from the origin to a distance of 9 feet.

a) 1 ft-lb b) 10 ft-lbs c) 9 ft-lbs d) 11 ft-lbs e) None of these