

LEVEL 3 (MATH ANALYSIS) BLITZ – 2007

- The value of $\sin^2 x + \cos^2 x$ is always
 - less than 1
 - equal to 1
 - more than 1
 - between -2 and 2
 - none of these
- The ratio of a to b is $4:3$. The sum of a and b is 14 . What is the ratio of $a - b$ to $a + b$?
 - $\frac{1}{3}$
 - $\frac{1}{5}$
 - $\frac{1}{7}$
 - $\frac{1}{9}$
 - none of these
- Find the value of $\cos(-31^\circ)$ if $\cos(46^\circ) = a$, $\sin(46^\circ) = b$, $\cos(77^\circ) = c$ and $\sin(77^\circ) = d$.
 - $ab + cd$
 - $ac + bd$
 - $ab - cd$
 - $ac - bd$
 - none of these
- The polar equation $r = 2 - 3 \cos \theta$ represents a
 - parabola
 - lemniscate
 - rose curve
 - limacon
 - none of these
- A ball is dropped from a height of h feet. Each time it strikes the ground, it rebounds to $\frac{3}{4}$ of the previous height. The total distance in feet traveled by the ball is:
 - $7h$
 - $6h$
 - $5h$
 - $4h$
 - none of these
- An expression that is equivalent to $\sin(4x) + \sin(2x)$ is:
 - $2 \sin 6x \sin 2x$
 - $2 \sin 3x \cos x$
 - $2 \cos 3x \sin x$
 - $2 \cos 6x \sin 2x$
 - none of these
- The equation $\sin 2x = \sin x$ has the following number of solutions in the interval $[0, 2\pi]$:
 - 1
 - 2
 - 3
 - 4
 - none of these

8. A manufacturer of widgets has found that, when the price of a widget is set at p dollars, the quantity sold is $75 - 5p$ where $0 \leq p \leq 15$. At what value should the price be set in order to maximize revenue?

- a) \$2.25 b) \$5.00 c) \$8.75 d) \$10.50 e) none of these

9. The trigonometric form of the vector $i - j$ is :

a) $\sqrt{2}((\cos 45^\circ)i + (\sin 45^\circ)j)$ b) $-\sqrt{2}((\cos 45^\circ)i - (\sin 45^\circ)j)$

c) $\sqrt{2}((\cos 135^\circ)i + (\sin 135^\circ)j)$ d) $\sqrt{2}((\cos 315^\circ)i + (\sin 315^\circ)j)$

- e) none of these

10. The expression $\sin(\text{Arc tan}(x) - \text{Arc cos}(y))$ is equivalent to:

a) $\frac{xy + \sqrt{1-y^2}}{\sqrt{1+x^2}}$ b) $\frac{xy - \sqrt{1-y^2}}{\sqrt{1+x^2}}$ c) $\frac{y + x\sqrt{1-y^2}}{\sqrt{1+x^2}}$ d) $\frac{y - x\sqrt{1-y^2}}{\sqrt{1+x^2}}$

- e) none of these

11. The lengths of the three sides of a triangle are 30 meters, 20 meters, and 14 meters. The area (in square meters) of the triangle is:

- a) $48\sqrt{6}$ b) $160\sqrt{77}$ c) 300 d) $20\sqrt{42}$ e) none of these

12. If $\|x\|$ = the greatest integer less than or equal to x , then the range of $f(x) = \|x\| + \|-x\|$ is :

- a) the set of all integers b) $[0, 1]$ c) $\{-1, 0\}$ d) $[-1, 1]$
e) none of these

13. A lawnmower blade with a 20-inch diameter is rotating at 3000 revolutions per minute. What is the linear velocity in inches per second of a tip of the blade?

- a) 600 b) 1000π c) 1000 d) 300π e) none of these

14. $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) =$

- a) $\frac{11\pi}{6}$ b) $\frac{5\pi}{6}$ c) $-\frac{\pi}{6}$ d) $-\frac{\pi}{3}$ e) none of these

15. Simplify $\frac{\sin x + \sin 3x + \sin 5x}{\cos x + \cos 3x + \cos 5x}$:

- a) $\tan x + \tan 3x + \tan 5x$ b) $\tan 3x$ c) $\tan 9x$ d) $\tan\left(\frac{9x}{2}\right)$
e) none of these

16. The exact value of $\cos 75^\circ$ is:

- a) $\frac{\sqrt{6} - \sqrt{2}}{4}$ b) $\frac{\sqrt{6} + \sqrt{2}}{4}$ c) $\frac{\sqrt{3} - 2\sqrt{2}}{2}$ d) $\frac{\sqrt{3} + 2\sqrt{2}}{2}$

e) none of these

17. If $a > 1$, the range of the function $f(x) = -2a^{-x} + 1$ is:

- a) $(-\infty, -2)$ b) $(-\infty, 0)$ c) $(-\infty, 1)$ d) all the real numbers
e) none of these

18. The value of $\cos^2(-75^\circ) - \sin^2(-75^\circ)$ is:

- a) 1 b) -1 c) $\frac{\sqrt{3}}{2}$ d) $-\frac{\sqrt{3}}{2}$ e) none of these

19. How many different vertical arrangements are there of 10 flags if 4 are white, 3 are blue, 2 are green and 1 is red?

- a) 24 b) 126 c) 4800 d) 1260 e) none of these

20. Each of the letters of the word MATH is written on an index card and the cards are then shuffled. What is the probability that, when the cards are dealt out, they spell the word MATH?

- a) $\frac{1}{4}$ b) $\frac{1}{6}$ c) $\frac{1}{36}$ d) $\frac{1}{64}$ e) none of these

21. An ellipse is defined by $4x^2 + y^2 - 8x + 4y + 4 = 0$. The length of the minor axis is:

- a) $\frac{1}{4}$ b) $\frac{1}{2}$ c) 2 d) 4 e) none of these

22. The phase shift of $3\sin\left(2x + \frac{\pi}{3}\right)$ is:

- a) $\frac{\pi}{3}$ b) $-\frac{\pi}{3}$ c) $\frac{\pi}{6}$ d) $-\frac{\pi}{6}$ e) none of these

23. If $\begin{vmatrix} a & b & c \\ x & y & z \\ 1 & 2 & 3 \end{vmatrix} = k$, then $\begin{vmatrix} 3 & 6 & 9 \\ x+1 & y+2 & z+3 \\ 2x+a & 2y+b & 2z+c \end{vmatrix} =$

- a) $-3k$ b) $-k$ c) $2k+1$ d) $3k+3$ e) none of these

24. If $f(x) = \ln(-x)$, then f is decreasing on the interval:

- a) $(-\infty, 0)$ b) $[-1, 0)$ c) $(0, 1]$ d) $(0, \infty)$ e) none of these

25. $\left(\sin\frac{\pi}{8} - \cos\frac{\pi}{8}\right)\left(\sin\frac{\pi}{8} + \cos\frac{\pi}{8}\right) =$

- a) $-\frac{\sqrt{2}}{2}$ b) $\frac{\sqrt{3}}{2}$ c) 1 d) $\frac{\sqrt{2}}{4}$ e) none of these

26. The coefficient of x^2 in the expansion of $\left(\sqrt{x} + \frac{3}{\sqrt{x}}\right)^8$ is:

- a) 28 b) 144 c) 252 d) 30 e) none of these

27. The sum of the solutions to the equation $\log_2 x^{\log_2 x} = 4$ is:

- a) $\frac{3}{2}$ b) $\frac{17}{4}$ c) $\frac{3}{8}$ d) $\frac{17}{32}$ e) none of these

28. The exact value of $\tan\left(\frac{175\pi}{3}\right)$ is :

- a) $\sqrt{3}$ b) 1 c) $\frac{\sqrt{3}}{3}$ d) $-\sqrt{3}$ e) none of these

29. A drawer full of socks contains $\frac{2}{3}$ white socks and $\frac{1}{3}$ black socks. Exactly $\frac{1}{2}$ of the white socks have a hole, whereas $\frac{2}{3}$ of the black socks have a hole. If a sock is selected at random and it has a hole, what is the probability that it was white?

- a) $\frac{1}{3}$ b) $\frac{3}{5}$ c) $\frac{5}{9}$ d) $\frac{7}{12}$ e) none of these

30. If $i = \sqrt{-1}$, then $\frac{1-i}{1+i} =$

- a) 0 b) -1 c) $2i$ d) $-i$ e) none of these

31. One of the foci of the hyperbola defined by $(x+1)^2 - (y+2)^2 = 4$ is:

- a) $(2\sqrt{2}-1, -2)$ b) $(2\sqrt{2}, -2)$ c) $(-1, 2\sqrt{2}-2)$ d) $(-1, 2\sqrt{2})$
e) none of these

32. In a circle of radius 18 miles, a sector has an interior angle of 10° . The length of the arc (in miles) of the sector is:

- a) 180 b) $\frac{129600}{17}$ c) π d) 1 e) none of these

33. The product of the non-real roots of $f(x) = x^3 - 8x^2 + 25x - 26$ is:

- a) 1 b) 2 c) 4 d) 13 e) none of these

34. If $f(x) = \log_3(4x-7)$, then $f^{-1}(2) =$

- a) 0 b) 1 c) 2 d) 4 e) none of these

35. One of the terms of the partial fraction decomposition of $\frac{10x^2 + 2x}{(x-1)^2(x^2+2)}$ has a numerator of:

- a) 4 b) 3 c) 2 d) 1 e) none of these

36. In a group of m men and w women, each man shakes hands with every other man and each woman shakes hands with every other woman. How many handshakes take place?

a) $\frac{mw}{2}$ b) $\frac{m^2 - m + w^2 - w}{2}$ c) $\frac{m^2 - w^2}{2}$ d) $\frac{m^2 + 2mw + w^2}{2}$

e) none of these

37. $\tan(\arccos \frac{\sqrt{3}}{2}) =$

a) $-\frac{\sqrt{3}}{3}$ b) $\frac{\sqrt{3}}{2}$ c) $\frac{\sqrt{3}}{3}$ d) $\sqrt{3}$ e) none of these

38. The equation of an asymptote of the graph of $y = \frac{x^3 + 1}{x^2 + 2x}$ is:

a) $y = 0$ b) $y = -2$ c) $y = x$ d) $y = x - 2$

e) none of these

39. In the x - y plane, the line segment with endpoints $(-5, 0)$ and $(25, 0)$ is the diameter of a circle. If the point $(x, 15)$ is on the circle, then $x =$

a) 5 b) 10 c) 15 d) 20 e) none of these

40. How many odd numbers greater than 40,000 can be formed by choosing from the digits 1, 2, 3, 4 and 5 without repeating any of these digits?

a) 10 b) 30 c) 60 d) 85 e) none of these

41. If $f(2x) = \frac{2}{2+x}$ for $x > 0$, then $2f(x) =$

a) $\frac{4}{4+x}$ b) $\frac{8}{4+x}$ c) $\frac{4}{2+x}$ d) $\frac{2}{2+x}$ e) none of these

42. If $\log 80 = A$ and $\log 45 = B$, then $\log 36 =$

- a) $A - 2B + 1$ b) $\frac{A+B}{2}$ c) $2A + B$ d) $A + B - 2$

e) none of these

43. The focus of the parabola described by $y^2 - 4y - 8x = 4$ is:

- a) (2, 0) b) (2, 1) c) (1, 2) d) (-1, 2)

e) none of these

44. The solutions of $2\sin^2 x - \sin x = 0$ in the interval $[0, 2\pi)$ are

- a) $\left\{0, \frac{\pi}{6}, \pi\right\}$ b) $\left\{\frac{\pi}{6}, -\frac{5\pi}{6}, \pi\right\}$ c) $\left\{0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi\right\}$

- d) $\left\{0, -\frac{\pi}{6}, -\frac{5\pi}{6}, \pi\right\}$ e) none of these

45. What is the smallest positive value of x , in radians, for which $\sin 7x = -\frac{1}{2}$

- a) $\frac{\pi}{6}$ b) $\frac{5\pi}{6}$ c) $\frac{\pi}{42}$ d) $\frac{11\pi}{42}$ e) none of these